

On the low energy spectra of the non-supersymmetric heterotic string theories

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Received: 9 January 2008 /

Published online: 16 February 2008 – © Springer-Verlag / Società Italiana di Fisica 2008

Abstract. The ten dimensional string theories as well as eleven dimensional supergravity are conjectured to arise as limits of a more basic theory, traditionally dubbed M-theory. This notion is confined to the ten dimensional supersymmetric theories. String theory, however, also contains ten dimensional non-supersymmetric theories that have not been incorporated into this picture. In this note we explore the possibility of generating the low energy spectra of various non-supersymmetric heterotic string vacua from the Horava–Witten model. We argue that this can be achieved by imposing on the Horava–Witten model an invariance with respect to some extra operators which identify the orbifold fixed planes in a non-trivial way, and we demonstrate it for the E_8 and $SO(16) \times SO(16)$ heterotic string vacua in ten dimensions.

1 Introduction

Superstring theory provides a consistent framework to explore the unification of gravity with the gauge interactions. The main feature of string theory is that it maintains the interpretation of the fundamental constituents of matter as elementary particles with internal quantum attributes. The string view of elementary particles is therefore a natural extension of the point particle interpretation, and may be regarded not to be distinct from it. The internal consistency of string theory imposes restrictions on the possible internal attributes. String theory facilitates the development of a phenomenological approach to quantum gravity, which is constrained by the string self-consistency requirements.

The internal consistency of string theory introduces the possibility of interpreting a number of degrees of freedom, required for consistency, as bosonic extra space-time dimensions. Thus, the bosonic string is formulated in twenty-six dimensions, while the fermionic strings are formulated in ten space-time dimensions. An important step in the development of string theory was obtained by the understanding that the five different supersymmetric ten dimensional theories may be related by perturbative and non-perturbative duality transformations [1, 2]. It was further suggested that the ten dimensional string theories can be related to eleven dimensional supergravity by compactification on S_1 and S_1/Z_2 [3–5]. In this regard the duality picture provides a compelling understanding of the ten dimensional string theories and indicates the existence of a more fundamental underlying structure.

The problem of supersymmetry breaking, and its realization in nature, is one of the vital issues in string theory. In addition to the supersymmetric ten dimensional fermionic string theories there exist ten dimensional fermionic string theories that are non-supersymmetric [6, 7]. These vacua have not been incorporated into the ten dimensional duality picture, although some preliminary studies were carried out in [8, 9]. It is likely that progress on understanding how to incorporate these non-supersymmetric vacua in the ten dimensional duality picture (see [10] for a recent discussion in the framework of the M(atr)ix theory), may be instrumental to progress on the problem of supersymmetry breaking in string theory.

In this paper we make an attempt in this direction. Our basic starting point is the eleven dimensional supergravity field theory compactified on S_1/Z_2 , à la Horava–Witten. We then study the possibility of reproducing the field theory content of the ten dimensional non-supersymmetric heterotic strings by imposing some extra symmetry operation on the Horava–Witten model. The basic ingredient in our analysis is the gluing of the end points of the Horava–Witten theory at $x^{10} = 0$ and $x^{10} = \pi$. However, the picture is further complicated due to the existence of the branes and the fields that reside on them. The essential point in our construction is an identification of the fields present on the boundary branes, located at the orbifold fixed points $x^{10} = 0$ and $x^{10} = \pi$. This operation is augmented by an extra operation, which interchanges the gauge degrees of freedom, in complete analogy with the one discussed in [7]. Identification of the degrees of freedom on the two different branes, located at the different orbifold fixed points, leads in turn to the identification of the branes themselves in order to maintain the locality of

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the low energy theory. The massless spectrum of the resulting models is obtained from the massless spectrum of the Horava–Witten model, by taking the x^{10} coordinate to be compact. In other words the operation we describe puts the orbifold fixed planes on top of each other with additional identification of the gauge degrees of freedom. This operation projects out certain fields from the initial spectrum (for example it projects out fermions from the supergravity sector in the first model) and in order to cancel the anomalies one has to add fermions with certain chirality on the resulting brane, in analogy with the Horava–Witten construction. In this manner we reproduce the massless spectrum of various non-supersymmetric ten dimensional heterotic string vacua from the Horava–Witten model, by an operation that essentially changes the geometry of the latter.

In the first example we investigate how to reproduce the spectrum of the E_8 Kac–Moody level two model. We propose that this can be achieved by imposing that the two fixed branes of the Horava–Witten model are identified by the orbifold condition, and imposing that the $E_8 \otimes E_8$ gauge group is broken to the diagonal E_8 level two subgroup. Anomaly cancellation is satisfied by adding a set of fermion fields from the twisted sector, as in the ten dimensional heterotic string vacuum. In the second example we discuss a model where the entire part of the $N = 1$ $D = 10$ supergravity multiplet is kept intact. As for the previous example, the model is anomalous unless we add extra massless fermions to the spectrum. In the final example we impose some extra operators onto the spectrum of the Horava–Witten model to obtain the massless spectrum of the non-supersymmetric $SO(16) \otimes SO(16)$ heterotic string model in a way similar to the previous two models.

2 Basic definitions

2.1 11 dimensional supergravity on S^1/Z_2

First let us collect the results of [4, 5]. This system corresponds to the effective theory of the strongly coupled $E_8 \otimes E_8$ heterotic superstring. It is described by eleven dimensional supergravity when one dimension, x^{10} , is compactified on the S^1/Z_2 orbifold. In this construction there are two ten dimensional “mirror” branes located at the orbifold fixed points $x^{10} = 0$ and $x^{10} = \pi$. The field content in the eleven dimensional bulk is the one of the usual eleven dimensional supergravity, i.e., it consists of the graviton G_{MN} , the gravitino Ψ_M and the antisymmetric tensor field of the third rank C_{MNP} , $M, N, P = 0, \dots, 10$. In order to obtain the field content on the orbifold fixed planes one assigns various eigenvalues to the fields under consideration with respect to the Z_2 parity operator \mathcal{R} . We have the following action of the parity operator on bosons:

$$\begin{aligned} \mathcal{R}G_{\mu\nu} &= G_{\mu\nu}, & \mathcal{R}C_{\mu\nu\rho} &= -C_{\mu\nu\rho}, & \mathcal{R}G_{\mu 10} &= -G_{\mu 10}, \\ \mathcal{R}G_{1010} &= G_{1010}, & \mathcal{R}C_{\mu\nu 10} &= C_{\mu\nu 10}, & \mu, \nu, \rho &= 0, \dots, 9, \end{aligned} \quad (1)$$

and for fermions

$$\begin{aligned} \mathcal{R}\psi_{\mu\alpha} &= \psi_{\mu\alpha}, & \mathcal{R}\psi_{\mu\dot{\alpha}} &= -\psi_{\mu\dot{\alpha}}, & \mathcal{R}\psi_{10\alpha} &= -\psi_{\mu 10}, \\ \mathcal{R}\psi_{10\dot{\alpha}} &= \psi_{10\dot{\alpha}}. \end{aligned} \quad (2)$$

At the orbifold fixed planes only fields that are even under the action of the parity group survive, and therefore we have a field content of $N = 1$ $D = 10$ supergravity i.e., bosonic fields $G_{\mu\nu}, C_{\mu\nu 10} \equiv B_{\mu\nu}, G_{1010} \equiv \phi$ and fermionic fields $\psi_{\mu\alpha}, \psi_{\dot{\alpha}}$. However, since the theory is chiral we need extra fermionic degrees of freedom to cancel the anomaly. This is achieved by adding two Yang–Mills multiplets $A_{\mu}^{1,2}, \xi_{\alpha}^{1,2}$, where each multiplet is located on different orbifold planes and belongs to adjoint representations of different E_8 gauge groups.

2.2 E_8 level two heterotic string model

Let us consider in some detail the spectrum of E_8 heterotic string theory [6, 7, 11, 12] realized at the Kac–Moody level two. This model can be obtained from the $E_8 \otimes E_8$ heterotic superstring by orbifolding it with the operator $\mathcal{M} = \mathcal{P} \otimes \mathcal{Q}$. Here the operator $\mathcal{P} = \exp(2\pi i J_{12})$ is the rotation operator. With respect to this operator bosons have eigenvalue $+1$ and fermions have eigenvalue -1 . The operator \mathcal{Q} interchanges the gauge degrees of freedom, which we denote as F^I and \tilde{F}^I .

The spectrum of the usual $E_8 \otimes E_8$ heterotic string is generated by oscillators that belong to the left and right moving sectors. In the right moving sector we have the oscillators of the ordinary closed superstring,

$$\begin{aligned} [\alpha_m^\mu, \alpha_n^\nu] &= -m\eta^{\mu\nu} \delta_{mn}, & \{d_m^\mu, d_n^\nu\} &= -\eta^{\mu\nu} \delta_{mn}, \\ \{b_m^\mu, b_n^\nu\} &= -\eta^{\mu\nu} \delta_{mn}. \end{aligned} \quad (3)$$

The mass formulas have the following form in the Ramond sector:

$$\frac{1}{4}M_R = \sum_{n=1}^{\infty} \alpha_{-n}^i \alpha_n^i + \sum_{n=1}^{\infty} n d_{-n}^i d_n^i, \quad (4)$$

and in the Neveu–Schwarz sector

$$\frac{1}{4}M_R = \sum_{n=1}^{\infty} \alpha_{-n}^i \alpha_n^i + \sum_{r=1/2}^{\infty} r b_{-r}^i b_r^i - \frac{1}{2}. \quad (5)$$

In the left moving sector we have oscillators

$$[\tilde{\alpha}_m^\mu, \tilde{\alpha}_n^\nu] = -m\eta^{\mu\nu} \delta_{mn}, \quad [\tilde{\alpha}_m^I, \tilde{\alpha}_n^J] = -m\delta^{IJ} \delta_{mn}, \quad (6)$$

where $I, J = 1, \dots, 16$ and the corresponding mass formula,

$$\frac{1}{4}M_L = \sum_{n=1}^{\infty} (\tilde{\alpha}_{-n}^i \tilde{\alpha}_n^i + \tilde{\alpha}_{-n}^I \tilde{\alpha}_n^I) + \sum_{I=1}^{16} (p_L^I)^2 - 1. \quad (7)$$

After orbifolding by the operator \mathcal{M} one obtains the following massless spectrum: in the untwisted sector we have fields $G_{\mu\nu}, B_{\mu\nu}, \phi$ generated via

$$b_{-1/2}^i |0\rangle_R \otimes \tilde{\alpha}_{-1/2}^j |0\rangle_L, \quad (8)$$

since $J_{12}b_{-1/2}^i|0\rangle_R = b_{-1/2}^i|0\rangle_R$, $J_{12}\alpha_{-1/2}^j|0\rangle_L = \alpha_{-1/2}^j|0\rangle_L$ and there are no gauge factors. The corresponding superpartners when the right vacuum is in the Ramond sector,

$$|0\rangle_R \otimes \tilde{\alpha}_{-1/2}^j|0\rangle_L, \tag{9}$$

are projected out, since they are not invariant under $\mathcal{P} \otimes \mathcal{Q}$ (since $J_{12}|0\rangle_R = -|0\rangle_R$ in the Ramond sector).

There are some more states in the untwisted sector, namely space-time bosons from the right sector should combine with the states in the left sector that are symmetric with respect to \mathcal{Q} , whereas the space-time fermions combine with the states that are antisymmetric with respect to \mathcal{Q} . The corresponding states are denoted by

$$b_{-1/2}^i|0\rangle_R \otimes |p^I\rangle_L \tag{10}$$

and

$$|0\rangle_R \otimes |\tilde{p}^I\rangle_L. \tag{11}$$

Thus we have 248 gauge bosons and fermions A_μ^i, ξ_α^i . They belong to the adjoint representation of the E_8 group realized at the Kac–Moody level two. This fact can be seen from the following considerations [11, 13–15]. In the heterotic string theory Cartan generators of $E_8 \otimes E_8$ group are realized in terms of

$$H_1^I = \frac{i}{\sqrt{2}}\partial F^i \quad \text{and} \quad H_1^I = \frac{i}{\sqrt{2}}\partial F^i.$$

From the OPE

$$F(z)F(w) = -\ln|z-w|^2$$

one can obtain

$$H_i^I(z)H_j^J(w) = \frac{1/2\delta_{ij}\delta^{IJ}}{(z-w)^2}.$$

The latter relation means that the gauge group is realized at level one. However, if we take as a diagonal subgroup $H^I = H_1^I + H_2^I$ and $E^a = E_1^a + E_2^a$, then computing the OPE between the Cartan generators we obtain

$$H_i(z)H_j(w) = \frac{\delta_{IJ}}{(z-w)^2},$$

which means that the gauge group is realized at level two Kac–Moody algebra. This in turn means that the roots of the algebra became shorter in accordance with the mass formula and the states just obtained are massless.

As is well known, in order to obtain a modular invariant partition function, one has to add the twisted sector to the model, i.e. one should consider the conditions

$$\begin{aligned} F^I(\sigma + \pi - t) &= \tilde{F}^I(\sigma - t) + \pi W^I, \\ \tilde{F}^I(\sigma + \pi - t) &= \tilde{F}^I(\sigma - t) + \pi \tilde{W}^I, \end{aligned} \tag{12}$$

where W^I and \tilde{W}^I are vectors in the E_8 lattice. The solution of these equations,

$$\begin{aligned} F^I(\sigma - t) &= F_0^I + M^I(\sigma - t) + \sum_r \frac{f_r^I}{r} e^{-2i(\sigma-t)}, \\ \tilde{F}^I(\sigma - t) &= \tilde{F}_0^I + \tilde{M}^I(\sigma - t) + \sum_r \frac{\tilde{f}_r^I}{r} e^{-2ir(\sigma-t)}, \end{aligned} \tag{13}$$

implies the relation between the oscillator modes

$$f_r^I = e^{2\pi ir} \tilde{f}_r^I = e^{4\pi ir} f_r^I, \tag{14}$$

which in turn means that r can be either integer or half-integer. The quantization leads in the massless twisted sector to the fermions $\tilde{\xi}_\alpha$ with the opposite chirality (being in the adjoint representation of E_8) to the ones in the untwisted sector and to a tachyon.¹

2.3 $SO(16) \otimes SO(16)$ heterotic string model

Let us discuss now the case of $SO(16) \otimes SO(16)$ heterotic string theory [6, 7]. The model can be obtained from the $E_8 \otimes E_8$ supersymmetric heterotic string theory by orbifolding it with the operator $\mathcal{F} = \mathcal{P} \otimes \mathcal{Q}$. Here again the operator $\mathcal{P} = \exp(2\pi i J_{12})$ is the rotation operator, while the operator \mathcal{Q} is an element of the group $E_8 \otimes E_8$ with the property $(\mathcal{Q})^2 = 1$. This element has the form $\mathcal{Q} = \mathcal{Q}_1 \otimes \mathcal{Q}_2$, where the element \mathcal{Q}_1 belongs to the center of the group $SO(16)$, which is the subgroup of E_8 . Under the action of this element the representation 120 of the group $SO(16)$ is even, while the representation 128 is odd. The element \mathcal{Q}_2 has completely the same properties with respect to the second group $SO(16)$. The spectrum of the theory contains the following massless fields: in the untwisted sector there are bosonic fields from the $N = 1$ ten dimensional supergravity multiplet $G_{\mu\nu}, B_{\mu\nu}, \phi$ and a gauge boson A_μ that belongs to the adjoint representation $(120, 1) \oplus (1, 120)$ of the group $SO(16) \otimes SO(16)$. In the fermionic sector one has fermions ψ_α that belong to the $(128, 1) \oplus (1, 128)$ representation of the group $SO(16) \otimes SO(16)$. In the massless part of the twisted sector one has just fermions $\tilde{\xi}_\alpha$ that belong to the $(16, 16)$ representation. The model is non-supersymmetric and chiral but anomaly free, since the number of the fermions with the opposite chirality is the same.

3 Models

3.1 Model 1

In this section we explore the possibility of obtaining from the Horava–Witten model the massless spectrum of the

¹ This can easily be seen from the mass formulae in the twisted sector for the left movers, $\frac{M_L^2}{8} = N^2 + p^2 - \frac{1}{2}$, where N can take integer and half-integer values and for the right movers in the Ramond sector, $\frac{M_R^2}{8} = N$, and in the Neveu–Schwarz sector, $\frac{M_S^2}{8} = N - \frac{1}{2}$, with N integer.

ten dimensional E_8 level two heterotic string model, described in Sect. 2.2. A natural way to achieve this goal is to require the invariance of the spectrum of the Horava–Witten model with respect to some extra operation, which acts on the gauge degrees of freedom. As discussed in Sect. 2.2 the operation of interchanging of the gauge degrees of freedom of a Lie group $G \otimes G$ breaks the gauge symmetry of the model to the diagonal subgroup G realized at level two Kac–Moody algebra. Besides that, the operation should identify the orbifold fixed planes in order to obtain the desired gauge symmetry on a single brane.

Following these considerations let us introduce two operators \mathcal{M} and \mathcal{N} and require the invariance of the spectrum of the Horava–Witten model under their action. The operator $\mathcal{M} = \mathcal{P} \otimes \mathcal{Q}$ is the one discussed in Sect. 2.2, while the operator \mathcal{N} acts only on the fields at the orbifold fixed planes and has the following properties:

$$\begin{aligned} \mathcal{N}G_{\mu\nu}(x^\mu, x^{10} = 0) &= G_{\mu\nu}(x^\mu, x^{10} = \pi), \\ \mathcal{N}B_{\mu\nu}(x^\mu, x^{10} = 0) &= B_{\mu\nu}(x^\mu, x^{10} = \pi), \\ \mathcal{N}\phi(x^\mu, x^{10} = 0) &= \phi(x^\mu, x^{10} = \pi), \\ \mathcal{N}G_{\mu\nu}(x^\mu, x^{10} = \pi) &= G_{\mu\nu}(x^\mu, x^{10} = 0), \\ \mathcal{N}B_{\mu\nu}(x^\mu, x^{10} = \pi) &= B_{\mu\nu}(x^\mu, x^{10} = 0), \\ \mathcal{N}\phi(x^\mu, x^{10} = \pi) &= \phi(x^\mu, x^{10} = 0), \end{aligned} \quad (15)$$

$$\begin{aligned} \mathcal{N}\psi_{\mu\alpha}(x^\mu, x^{10} = 0) &= \psi_{\mu\alpha}(x^\mu, x^{10} = \pi), \\ \mathcal{N}\psi_{\dot{\alpha}}(x^\mu, x^{10} = 0) &= \psi_{\dot{\alpha}}(x^\mu, x^{10} = \pi), \\ \mathcal{N}\psi_{\mu\alpha}(x^\mu, x^{10} = \pi) &= \psi_{\mu\alpha}(x^\mu, x^{10} = 0), \\ \mathcal{N}\psi_{\dot{\alpha}}(x^\mu, x^{10} = \pi) &= \psi_{\dot{\alpha}}(x^\mu, x^{10} = 0). \end{aligned} \quad (16)$$

$$\begin{aligned} \mathcal{N}A_\mu^1(x^\mu, x^{10} = 0) &= A_\mu^2(x^\mu, x^{10} = \pi), \\ \mathcal{N}\xi_\alpha^1(x^\mu, x^{10} = 0) &= \xi_\alpha^2(x^\mu, x^{10} = \pi), \\ \mathcal{N}A_\mu^2(x^\mu, x^{10} = \pi) &= A_\mu^1(x^\mu, x^{10} = 0), \\ \mathcal{N}\xi_\alpha^2(x^\mu, x^{10} = \pi) &= \xi_\alpha^1(x^\mu, x^{10} = 0). \end{aligned} \quad (17)$$

The requirement of the invariance of the spectrum of the Horava–Witten model under the action of the operators \mathcal{M} and \mathcal{N} leads to the following massless fields. First consider the case of gauge bosons $A_\mu^{1,I}(x^\mu, x^{10} = 0)J^I$ and $A_\mu^{2,I}(x^\mu, x^{10} = \pi)\tilde{J}^I$, where J^I and \tilde{J}^I are generators of the first and the second E_8 groups, respectively. Making use of the operator \mathcal{N} we can introduce a combination

$$\begin{aligned} \mathcal{N}A_\mu^{1,I}(x^\mu, x^{10} = 0)J^I + A_\mu^{2,I}(x^\mu, x^{10} = \pi)\tilde{J}^I \\ = A_\mu^{2,I}(x^\mu, x^{10} = \pi)(J^I + \tilde{J}^I). \end{aligned} \quad (18)$$

The right hand side is obviously invariant with respect to the operator \mathcal{M} . A requirement of the invariance with respect to the operator \mathcal{N} gives the further constraint $A_\mu^{1,I}(x^\mu, x^{10} = 0) = A_\mu^{2,I}(x^\mu, x^{10} = \pi)$. Therefore, one obtains a single gauge field on a brane (two initial orbifold planes are effectively identified) and the E_8 gauge theory is realized at the level two Kac–Moody algebra. For the fields $G_{\mu\nu}, B_{\mu\nu}$ and ϕ the discussion is completely the same, namely after imposing the invariance under the op-

erators just introduced only the symmetric combination of these fields survives.

For the fermionic fields from the Yang–Mills supermultiplet the situation is again similar with a slight modification. Namely, since they are odd with respect to the operator \mathcal{J} , the combination that is invariant under the action of \mathcal{M} has the form

$$\begin{aligned} \mathcal{N}\xi_\alpha^{1,I}(x^\mu, x^{10} = 0)J^I - \xi_\alpha^{2,I}(x^\mu, x^{10} = \pi)\tilde{J}^I \\ = \xi_\alpha^{2,I}(x^\mu, x^{10} = \pi)(J^I - \tilde{J}^I). \end{aligned} \quad (19)$$

Finally the fermions $\psi_{\mu\alpha}, \psi_{\dot{\alpha}}$ from the ten dimensional supergravity multiplet are projected out, since they are not invariant under the action of the operator \mathcal{M} . Therefore requiring the invariance of the spectrum under the action of the operators \mathcal{M} and \mathcal{N} projects out the fermionic sector of 10D supergravity, while the rest of the fields gives rise to the untwisted sector of the E_8 model.

The theory as it stands is anomalous, since it is chiral. In order to cancel the anomaly one has to add the same number of fermions but with the opposite chirality; let us denote them $\tilde{\xi}_{\dot{\alpha}}$. Since the orbifold fixed planes are now effectively identified we need to impose the “proper” transformation properties with respect to the operator \mathcal{M} . This means that the fermions belong to the adjoint representation of the gauge group E_8 and have the form $\tilde{\xi}_{\dot{\alpha}}^I(J^I - \tilde{J}^I)$. These fermions are analogous to those that appear in the twisted sector of the E_8 level two heterotic string model, thus leading to the absence of anomalies and to its modular invariance.

Finally, let us note that the ten dimensional E_8 heterotic string theory is tachyonic, whereas the eleven dimensional orbifold that we considered is non-supersymmetric and tachyon free. One possibility is to simply add a tachyonic field to the spectrum. A more interesting possibility is the case that the tachyon becomes massive at the limit we are considering. We note that such a decoupling of the tachyon as a function of moduli is a well known phenomenon in string vacua [16–19].

3.2 Model 2

Another possibility is to consider a model when one keeps the entire supergravity multiplet intact. For this reason one has to assign non-trivial transformation properties under the operator \mathcal{N} to the fermions $\psi_{\mu\alpha}, \psi_{\dot{\alpha}}$ from the $N = 1$ ten dimensional supergravity multiplet in analogy with (15)–(17). However, in order to keep these fermions unprojected this is not enough and one has to modify the form of the operator \mathcal{M} as well, since all fermionic degrees of freedom in the initial Horava–Witten model were odd with respect to the operator \mathcal{J} . The simplest choice is to take $\mathcal{M} = \mathcal{Q}$ for this model. The transformation properties under the action of \mathcal{N} for the other fields are again (15)–(17). The discussion of the spectrum is now completely analogous to the previous model. Namely, in the Yang–Mills sector of the model we again have a ten dimensional Yang–Mills supermultiplet with the gauge group E_8 realized at the Kac–Moody level two, and its fermionic part

is obtained by taking

$$\begin{aligned} \mathcal{N}\xi_\alpha^{1,I}(x^\mu, x^{10} = 0)J^I + \xi_\alpha^{2,I}(x^\mu, x^{10} = \pi)\tilde{J}^I \\ = \xi_\alpha^{2,I}(x^\mu, x^{10} = \pi)(J^I + \tilde{J}^I). \end{aligned} \quad (20)$$

On the supergravity side we have a complete ten dimensional $N = 1$ supergravity multiplet. As in the previous model, the model is chiral, since, though the supergravity multiplet has been kept, the number of fermionic degrees of freedom in the super Yang–Mills multiplet has been halved. Therefore, in order to cancel the anomalies, one has to add the extra fermions $\tilde{\xi}_\alpha^I(J^I + \tilde{J}^I)$, with the same chirality as the ones from the Yang–Mills supermultiplet, in the adjoint representation of the gauge group.

To summarize, this model is quite similar to the Horava–Witten model but with the gauge symmetry realized at the Kac–Moody level two.

3.3 Model 3

Finally, let us discuss how one can obtain the massless spectrum of the non-tachyonic $SO(16) \otimes SO(16)$ heterotic string from the Horava–Witten model by imposing on it some extra symmetries. Similarly to the previous examples this extra symmetry actually coincides with the one imposed on the $E_8 \otimes E_8$ heterotic string to obtain the ten dimensional $SO(16) \otimes SO(16)$ model, but enhanced by the extra operator \mathcal{N} . Therefore, the symmetry under consideration is generated by operators \mathcal{F} and \mathcal{N} , where the operator \mathcal{F} is defined in Sect. 2.3. Requiring invariance of the spectrum of the Horava–Witten model with respect to the action of \mathcal{F} projects out the fermions $\psi_{\mu\alpha}, \psi_{\dot{\alpha}}$ from the supergravity spectrum and leads to the massless fields that belong to the massless untwisted sector of the ten dimensional $SO(16) \otimes SO(16)$ model. The action of the operator \mathcal{N} on the bosonic fields from the ten dimensional supergravity multiplet is the same as (15), while for the field from the Yang–Mills supermultiplet it reads

$$\begin{aligned} \mathcal{N}A_\mu^1(x^\mu, x^{10} = 0) &= A_\mu^1(x^\mu, x^{10} = \pi), \\ \mathcal{N}\xi_\alpha^1(x^\mu, x^{10} = 0) &= \xi_\alpha^1(x^\mu, x^{10} = \pi), \\ \mathcal{N}A_\mu^2(x^\mu, x^{10} = \pi) &= A_\mu^2(x^\mu, x^{10} = 0), \\ \mathcal{N}\xi_\alpha^2(x^\mu, x^{10} = \pi) &= \xi_\alpha^2(x^\mu, x^{10} = 0), \end{aligned} \quad (21)$$

which simply means that we are putting the orbifold branes on top of each other. This model is again anomalous, since as in the previous models we are missing massless fermions from the twisted sector. Therefore, to cancel the anomalies we add an extra set of fermions $\tilde{\xi}_\alpha$ with the chirality opposite to the ones of the fermions from the Yang–Mills supermultiplet. We can take these fermions to be in the $(16, 16)$ representation of the $SO(16) \otimes SO(16)$ gauge group, and this choice leads to the massless spectrum of the corresponding heterotic string model. This choice is not unique, however. Namely, one can take these extra fermions to be exactly in the same representation $(128, 1) \oplus (1, 128)$ of the gauge group as those from the Yang–Mills supermultiplet. In this way the model will be anomaly free, since it is non-chiral, but it has no string theory counterpart.

4 Conclusions

String theory provides an internally consistent framework for the phenomenological approach to quantum gravity. Important progress has been achieved in string theory with the realization that the five supersymmetric string theories in ten dimensions are related by perturbative and non-perturbative duality transformations. This understanding is, however, not complete, as the non-supersymmetric ten dimensional string theories are not incorporated in the duality picture. It is plausible that the understanding of some phenomenological issues in string theory is contingent on understanding how the non-supersymmetric string vacua fit into the duality picture. In this note we explored the possibility of generating the spectrum of the E_8 and $SO(16) \times SO(16)$ ten dimensional heterotic string theories as orbifolds of the Horava–Witten theory. We argued that this is indeed possible by imposing the appropriate identification conditions on the fields that reside on the two fixed branes. We further demonstrated the conditions required for the resulting theories to be anomaly free. One can contemplate that the generated orbifolds are the non-perturbative limits of the ten dimensional non-supersymmetric string theories and subject this hypothesis to further tests.

Our basic starting point is the eleven dimensional supergravity field theory compactified on S_1/Z_2 , à la Horava–Witten. The basic point in our analysis is the gluing of the end points of the Horava–Witten theory at $x^{10} = 0$ and $x^{10} = \pi$, and the identification of the fields present on the boundary branes, located at the orbifold fixed points $x^{10} = 0$ and $x^{10} = \pi$. This operation is augmented by an additional operation that acts on the gauge degrees of freedom, in analogy with the one discussed in [7]. Identification of the degrees of freedom on two different branes located at the different orbifold fixed points entails the identification of the branes themselves in order to maintain the locality of the low energy theory. The massless spectrum in these models is obtained from that of the Horava–Witten model by taking the eleventh coordinate to be compact. In other words, the operation entails putting the orbifold fixed planes on top of each other, with additional identification of gauge degrees of freedom. This operation projects out certain fields from the initial spectrum. Anomaly cancellation dictates the addition of fermionic fields with certain chirality on the resulting brane, in analogy with the Horava–Witten construction. We reproduced the massless spectra of various non-supersymmetric ten dimensional heterotic string vacua from the Horava–Witten model, by an operation that essentially changes the geometry of the latter. We remark that one can consider an alternative orbifold by moding the eleventh dimension by the reflection symmetry under the exchange $x_{10} \leftrightarrow \pi - x_{10}$. Under this orbifold there would be a fixed orbifold point at $x = \pi/2$. This operation differs from the one that we considered in this paper and may be of interest for further exploration.

Acknowledgements. We would like to thank Carlo Angelantonj, Nikos Irges, Elias Kiritsis and Thomas Mohaupt for

discussions. AEF would like to thank the INFN Galileo Galilei Institute in Florence for hospitality. This work was supported in part by UK Science and Technology Facilities Council under the grant PP/D000416/1 and by the “UniverseNet” MRTN-CT-2006-035863 and “Superstring Theory” MRTN-CT-2004-512194.

References

1. C.M. Hull, P.K. Townsend, Nucl. Phys. B **438**, 109 (1995) [hep-th/9410167]
2. E. Witten, Nucl. Phys. B **443**, 85 (1995) [hep-th/9503124]
3. M.J. Duff, P.S. Howe, T. Inami, K.S. Stelle, Phys. Lett. B **191**, 70 (1987)
4. P. Horava, E. Witten, Nucl. Phys. B **460**, 506 (1996) [hep-th/9510209]
5. P. Horava, E. Witten, Nucl. Phys. B **475**, 94 (1996) [hep-th/9603142]
6. H. Kawai, D.C. Lewellen, S.H.H. Tye, Phys. Rev. D **34**, 3794 (1986)
7. L.J. Dixon, J.A. Harvey, Nucl. Phys. B **274**, 93 (1986)
8. J.D. Blum, K.R. Dienes, Nucl. Phys. B **516**, 83 (1998) [hep-th/9707160]
9. O. Bergman, M.R. Gaberdiel, Nucl. Phys. B **499**, 183 (1997) [hep-th/9701137]
10. O. Aharony, Z. Komargodski, A. Patir, JHEP **0705**, 073 (2007) [hep-th/0702195]
11. A. Font, L.E. Ibanez, F. Quevedo, Nucl. Phys. B **345**, 389 (1990)
12. P. Forgacs, Z. Horvath, L. Palla, P. Vecsernyes, Nucl. Phys. B **308**, 477 (1988)
13. K.R. Dienes, J. March-Russell, Nucl. Phys. B **479**, 113 (1996) [hep-th/9604112]
14. G. Aldazabal, A. Font, L.E. Ibanez, A.M. Uranga, Nucl. Phys. B **452**, 3 (1995) [hep-th/9410206]
15. G. Aldazabal, A. Font, L.E. Ibanez, A.M. Uranga, Nucl. Phys. B **465**, 34 (1996) [hep-th/9508033]
16. P.H. Ginsparg, C. Vafa, Nucl. Phys. B **289**, 414 (1987)
17. V.P. Nair, A.D. Shapere, A. Strominger, F. Wilczek, Nucl. Phys. B **287**, 402 (1987)
18. C. Angelantonj, M. Cardella, N. Irges, Phys. Lett. B **641**, 474 (2006) [hep-th/0608022]
19. C. Angelantonj, M. Cardella, N. Irges, Nucl. Phys. B **725**, 115 (2005) [hep-th/0503179]